Frazioni particolari

Fazione	Valore
$\frac{n}{0}$	∞
$\frac{\infty}{0}$	∞
$\frac{n}{\infty}$	0
$\frac{0}{n}$	0
$\frac{0}{\infty}$	0

Potenze particolari

Potenza	Valore
0^n	0
n^0	1
n^∞	∞
$n^{-\infty}$	0
∞^0	$\emptyset$
0^∞	0

Operazioni tra potenze

Operazione	Valore
$a^m \cdot a^n$	a^{m+n}
$\frac{a^m}{a^n}$	a^{m-n}
$(a^m)^n$	$a^{m \cdot n}$
$(a^n \cdot b^n)$	$(a \cdot b)^n$
$\left(\frac{a^n}{b^n}\right)$	$\left(\frac{a}{b}\right)^n$

Equazioni parametriche curve fondamentali

Curva	Equazione	Eq. parametrica
Circonferenza	$x^2 + y^2 = r^2$ $c(0,0)\ \ t \in [0,2\pi]$	$\begin{cases} x(t) = cost \\ y(t) = sint \end{cases}$
Ellisse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $A(\pm a, 0)\ B(0, \pm b)$ $t \in [0,2\pi]$	$\begin{cases} x(t) = a\ cost \\ y(t) = b\ sint \end{cases}$

Formule di duplicazione

$sin\ 2\alpha = 2sin\ \alpha\ cos\ \alpha$

$cos\ 2\alpha = cos^2\alpha - sin^2\alpha = \begin{cases} 1 - 2sin^2\alpha \\ 2\ cos^2\alpha - 1 \end{cases}$

$tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}$ con $\alpha \neq 45° \wedge k90° + k180°$

Formule di bisezione

$cos^2\frac{x}{2} = \frac{1+cosx}{2} \Longrightarrow cos^2x = \frac{1+cos2x}{2}$

$sin^2\frac{x}{2} = \frac{1-cosx}{2} \Longrightarrow sin^2x = \frac{1-cos2x}{2}$

Formule di Werner

$sin\alpha\ sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$cos\alpha\ cos\beta = \frac{1}{2}[cos(\alpha + \beta) + cos(\alpha - \beta)]$
$sin\alpha\ cos\beta = \frac{1}{2}[sin(\alpha + \beta) + sin(\alpha - \beta)]$

Formule parametriche

$\sin\alpha = \frac{2t}{1+t}$ $\cos\alpha = \frac{1-t^2}{1+t^2}$ con $t = tg\frac{\alpha}{2}$ e $\alpha \neq 180° + k360°$

$tg\alpha = \frac{2t}{1-t^2}$ con $\mathrm{t} = \mathrm{tg}\frac{\alpha}{2}$ e $\alpha \neq 90° + k180° \wedge 90° + k180°$

Formule di Eulero

$$\begin{cases} \sin t = \frac{e^{jt} - e^{-jt}}{2j} \\ \cos t = \frac{e^{jt} + e^{-jt}}{2} \\ \sinh t = \frac{e^t - e^{-t}}{2} \\ \cosh t = \frac{e^t + e^{-t}}{2} \end{cases}$$

Formule di addizione e sottrazione

Funzione	ADDIZIONE	SOTTRAZIONE
Seno	$\sin(\alpha+\beta) = sin\alpha\ cos\beta + cos\alpha\ sin\beta$	$sin(\alpha-\beta) = sin\alpha\ cos\beta - cos\alpha\ sin\beta$
Coseno	$\cos(\alpha+\beta) = cos\alpha\ cos\beta - \sin\alpha\ \ sin\beta$	$\cos(\alpha-\beta) = cos\alpha\ cos\beta + \sin\alpha\ \ sin\beta$
Tangente	$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$	$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$
Cotangente	$\tan^{-1}(\alpha+\beta) = \dfrac{1+\tan^{-1}\alpha\tan^{-1}\beta}{\tan^{-1}\alpha+\tan^{-1}\beta}$	$\tan^{-1}(\alpha+\beta) = \dfrac{1-\tan^{-1}\alpha\tan^{-1}\beta}{\tan^{-1}\alpha-\tan^{-1}\beta}$

Angoli opposti, complementari e supplementari

- **Angoli opposti:**
  $cos(-\alpha) = cos(\alpha)$
  $sin(\alpha) = -sin(\alpha)$
- **Angoli complementari:**
  $cos\left(\frac{\pi}{2}-\alpha\right) = sin(\alpha) \quad \Rightarrow \quad cos\left(\alpha-\frac{\pi}{2}\right) = sin(\alpha)$
  $cos\left(\frac{\pi}{2}+\alpha\right) = -sin(\alpha)$
  $sin\left(\frac{\pi}{2}\pm\alpha\right) = cos(\alpha) \quad \Rightarrow \quad sin\left(\alpha-\frac{\pi}{2}\right) = -cos(\alpha)$
- **Angoli supplementari:**
  $cos(\pi\pm\alpha) = -cos(\alpha) \ \Rightarrow \ cos(\alpha-\pi) = -cos(\alpha)$
  $sin(\pi-\alpha) = sin(\alpha) \quad \Rightarrow \ sin(\alpha-\pi) = -sin(\alpha)$
  $sin(\pi+\alpha) = -sin(\alpha)$

Sostituzioni varie

$1-\sin^2 x = \cos^2 x$ $\qquad \sin^2 x + \cos^2 x = 1$ $\qquad \tan x = \frac{\sin x}{\cos x}$ $\qquad \frac{1}{\sin^2 x} = \text{cosec}^2 x$ $\qquad \frac{1}{\cos^2 x} = \sec^2 x$

Limiti notevoli

$\lim_{x\to 0}\frac{\sin x}{x}=1$	$\lim_{x\to 0}\frac{x}{\sin x}=1$	$\lim_{x\to 0}\frac{\sin^{n} x}{x^n}=1$
$\lim_{x\to 0}\frac{x^n}{\sin^{n} x}=1$	$\lim_{x\to 0}\frac{\sin kx}{kx}=1$	$\lim_{x\to 0}\frac{kx}{\sin kx}=1$
$\lim_{x\to 0}\frac{(1+x)^k-1}{x}=1$	$\lim_{x\to 0}\frac{a^x-1}{x}=\log a$	$\lim_{x\to 0}\frac{e^x-1}{x}=1$
$\lim_{x\to 0}\left(1+\frac{1}{x}\right)^x=e$	$\frac{\lim_{x\to 0}\left(\sqrt[n]{1+x}-1\right)}{x}=\frac{1}{n}$	$\lim_{x\to 0}\log\left(1+\frac{1}{x}\right)^x=1$
$\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$	$\lim_{x\to 0}\log(1+x)^{\frac{1}{x}}=1$	$\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\log_a e$
$\lim_{x\to 0}\tan x=0$	$lim_{x\to 0}\frac{\tan x}{x}=1$	$\lim_{x\to 0}\arccos x=\frac{\pi}{2}$
$\lim_{x\to 0}\frac{\arcsin x}{x}=1$	$\lim_{x\to 0}\arcsin x=0$	$\lim_{x\to 0}\frac{\arctan x}{x}=1$
$\lim_{x\to \infty}\left(1+\frac{1}{x}\right)^x=e$		

Principali polinomi di Taylor

$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+o(x^n)$

$\sin x=x-\frac{x^3}{6}+\frac{x^5}{5!}+\cdots+\frac{(-1)^n}{(2n+1)!}x^{(2n+1)}+o(x^{2n+2})$

$\cos x=1-\frac{x^2}{2}+\frac{x^4}{4!}+\cdots+\frac{(-1)^n}{(2n)!}x^{2n}+o(x^{2n+1})$

$\tan x=x+\frac{x^3}{3}+\frac{x^2}{15}x^5+\frac{17}{315}x^7+\frac{62}{2835}x^9+o(x^{10})$

$\frac{1}{1-x}=1+x+x^2+x^3+\cdots+x^n+o(x^n)$

$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^5}{5}+\cdots+\frac{(-1)^{n+1}}{n}x^n+o(x^n)$

$(1+x)^\alpha=1+\alpha x+\frac{\alpha(\alpha-1)}{2}x^2+\frac{\alpha(\alpha-1)(\alpha-2)}{6}x^3+\cdots+(\alpha\ n)x^n+o(x^n)$

Derivate elementari

Funzione	Derivata
$y = k$	$y' = 0$
$y = x$	$y' = 1$
$y = \frac{1}{x}$	$y' = -\frac{1}{x^2}$
$y = x^n$	$y' = nx^{n-1}$
$y = \sqrt[n]{x}$	$y' = \frac{1}{n\sqrt[n]{x^{n-1}}}$
$y = \sqrt{z}$	$y' = \frac{1}{2\sqrt{x}}$
$y = \sqrt[n]{x^m}$ $m < n$	$y' = \frac{m}{n\sqrt[n]{x^{n-m}}}$
$y = \log_a x$	$y' = \frac{1}{x}\log_a e$
$y = \log x$	$y' = \frac{1}{x}$
$y = e^x$	$y' = e^x$
$y = a^x$	$y' = a^x \log a$
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \frac{1}{\cos^2 x} = 1 + \tan^2 x = \sec^2 x$
$y = \text{cotan}\, x$	$y' = -\frac{1}{\sin^2 x} = -1 - \text{cotan}^2 x$
$y = \arcsin x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \arccos x$	$y' = -\frac{1}{\sqrt{1-x^2}}$
$y = \arctan x$	$y' = \frac{1}{1+x^2}$
$y = \text{arccotg}\, x$	$y' = -\frac{1}{1+x^2}$

$y = f(x) \cdot g(x) \quad \Rightarrow \quad y' = f'(x)g(x) + g'(x)f(x)$

$y = \frac{f(x)}{g(x)} \quad \Rightarrow \quad y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Integrali immediati

$\int 1 dx = x + c$

$\int \frac{1}{x} dx = \log|x| + c$

$\int e^x \ dx = e^x + c$

$\int e^{nx} \ dx = \frac{e^{nx}}{n} + c$

$\int a^x \, dx = \frac{1}{\log a} a^x + c$

$\int \sin x \ dx = -\cos x + c$

$\int \cos x \, dx = \sin x + c$

$\int \frac{1}{\cos^2 x} dx = \tan x + c$

$\int [1 + \tan^2 x] \, dx = \tan x + c$

$\int \frac{1}{\sin^2 x} dx = -\text{cotan} x + c$

$\int [-1 - \text{cotan}^2 x] dx = \text{cotan} x + c$

$\int \frac{1}{\sqrt{1+x^2}} dx = \arcsin x + c$

$\int \frac{1}{1+x^2} dx = \arctan x + c = -\text{arccotan} + c$

$\int \tan x \, dx = -\log(\cos x) + c$

$\int \text{cotan} x \, dx = \log(\sin x) + c$

$\int x \, dx = \frac{x^2}{2} + c$

$\int x^n \, dx = \frac{x^{n+1}}{n+1}$

Integrazione per parti $\rightarrow$ $\int f'(x) g(x) \, dx = f(x) g(x) - \int f(x) g'(x) \ dx.$

$\int \frac{f'(x)}{f(x)} dx = \log\big(f(x)\big) + c$

$\int e^{f(x)} \cdot f'(x) \, dx = e^{f(x)} + c$

$\int [f(x)]^a \cdot f(x) \, dx = \frac{[f(x)]^{a+1}}{a+1} + c$

$\int f(g(x)) \cdot g'(x) \, dx = \int f(t) \, dt$ con $t = g(x)$

Proprietà delle trasformate di Fourier e di Laplace

Proprietà	Segnale	Fourier	Laplace bilatera
Linearità	$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$	$aX(s) + bY(s)$
Traslazione nel tempo	$x(t - t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-st_0}X(s)$
Traslazione in ω	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$	$X(s - j\omega_0)$
Traslazione in s	$e^{s_0 t}x(t)$		$X(s - s_0)$
Riscaldamento	$x(at)$	$\left(\frac{1}{\lvert a\rvert}\right)X\left(\frac{\omega}{a}\right)$	$\left(\frac{1}{\lvert a\rvert}\right)X\left(\frac{s}{a}\right)$
Derivata nel tempo	$x^{'}(t)$	$j\omega X(\omega)$	$Sx(S)$
Derivate nel tempo	$x^{(n)}(t)$	$(j\omega)^n X(\omega)$	$s^n X(s)$
Derivata in ω	$-jtx(t)$	$X'(\omega)$	
Derivata in s	$-tx(t)$		$X'(s)$
Simmetria	$X(t)$	$2\pi x(-\omega)$	
Coniugazione	$x^*(t)$	$X^*(-\omega)$	$X^*(s^*)$
Realtà e parità	$x(t)$ reale e pari	$X(\omega)$ reale e pari	
Realtà e disparità	$x(t)$ reale e dispari	$X(\omega)$ immaginaria pura e dispari	$X(s - s_0)$
Convoluzione	$x(t) * y(t)$	$X(\omega)Y(\omega)$	$X(s)Y(s)$
Prodotto	$x(t)y(t)$	$\left(\frac{1}{2\pi}\right)X(\omega)Y(\omega)$	
$x(t)$ periodica di periodo $T = 2\pi/\omega_0$ $I = [t_o, t_0 + T[$ intervallo di ampiezza T	$x(t) = x(t - T)$ $x_0(t) = \begin{cases} x(t) & t \in I \\ 0 & t \notin I \end{cases}$	$X(\omega)$ $\sum_{n=-\infty}^{+\infty} \omega_0 X_0(n\omega)\delta(\omega - n\omega_0)$ $X_0(\omega) = \mathfrak{F}[x_0(t)]$	
$y(t) = \begin{cases} x(t) & t \geq I \\ 0 & t < I \end{cases}$ $x(t)$ periodica di periodo $T = 2\pi/\omega_0$	$y(t) = \sum_{n=0}^{+\infty} x_0(t - nT)$ $x(t) = x(t - T)$ $x_0(t) = \begin{cases} x(t) & 0 \leq t < T \\ 0 & t < 0 \\ 0 & t \geq T \end{cases}$		$Y(s) = \frac{X_0(s)}{1 - e^{-sT}}$ $X_0(s) = \mathcal{L}[x_0(t)]$ $X_0(s)$ analitica in $\mathbb{C}$

Trasformata $\mathfrak{F}$ e $\mathcal{L}$ di alcune funzioni e distribuzioni

$x(t)$	$\mathfrak{F}[x(t)] = X(\omega)$	$\mathcal{L}[x(t)] = X(s)$	$dom_{\mathcal{L}} X(s)$
$\delta(t)$	1	1	$\forall s$
$\delta(t-t_0)$	$e^{-j\omega t_0}$	$e^{-t_0 s}$	$\forall s$
$u(t+T)-u(t-T)$	$\frac{2sin\omega T}{\omega}$	$\frac{2sinhsT}{s}$	$\forall s$
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{1}{s}$	$\boldsymbol{Re}\, s > 0$
e^{-at^2} $a \in \mathbb{R}\ a > 0$	$\sqrt{\frac{\pi}{a}} e^{-\frac{\omega^2}{4a}}$	$\sqrt{\frac{\pi}{a}} e^{\frac{s^2}{4a}}$	$\forall s$
$\frac{n/\pi}{1+n^2t^2}$ $n \in \mathbb{N}$	$e^{\frac{-\lvert\omega\rvert}{n}}$	non esiste	
$u(t-t_0)$	$\pi\delta(\omega)+\frac{1}{j\omega}e^{-j\omega t_0}$	$\frac{1}{s}e^{-t_0 s}$	$\boldsymbol{Re}\, s > 0$
$u(t)e^{j\omega_0 t}$	$\pi\delta(\omega-\omega_0)+\frac{1}{j(\omega-\omega_0)}$	$\frac{1}{s-j\omega_0}$	$\boldsymbol{Re}\, s > 0$
$\mathrm{u(t)e^{s_0t}}$ $\boldsymbol{Re}\, s_0 < 0$	$\frac{1}{j\omega - s_0}$	$\frac{1}{s-s_0}$	$\boldsymbol{Re}\, s > \boldsymbol{Re}\, s_0$
$\mathrm{u(t)e^{s_0t}}$ $\boldsymbol{Re}\, s_0 > 0$	non esiste	$\frac{1}{s-s_0}$	$\boldsymbol{Re}\, s > \boldsymbol{Re}\, s_0$
$u(t)sin\omega_0 t$ $\omega_0 \in \mathbb{R}$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\boldsymbol{Re}\, s > 0$
$u(t)e^{\sigma_0 t}sin\omega_0 t$ $\sigma_0, \omega_0 \in \mathbb{R}\ \sigma_0 < 0$	$\frac{\omega_0}{(j\omega-\sigma_0)^2+\omega_0^2}$	$\frac{\omega_0}{[(s-\sigma_0)^2+\omega_0^2]}$	$\boldsymbol{Re}\, s > \sigma_0$
$u(t)e^{\sigma_0 t}sin\omega_0 t$ $\sigma_0, \omega_0 \in \mathbb{R}\ \sigma_0 > 0$	non esiste	$\frac{\omega_0}{[(s-\sigma_0)^2+\omega_0^2]}$	$\boldsymbol{Re}\, s > \sigma_0$
$u(t)cos\omega_0 t$ $\omega_0 \in \mathbb{R}$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	$\frac{s}{s^2+\omega_0^2}$	$\boldsymbol{Re}\, s > 0$
$u(t)e^{\sigma_0 t}cos\omega_0 t$ $\sigma_0, \omega_0 \in \mathbb{R}\ \sigma_0 < 0$	$\frac{j\omega-\sigma_0}{(j\omega-\sigma_0)^2+\omega_0^2}$	$\frac{s-\sigma_0}{[(s-\sigma_0)^2+\omega_0^2]}$	$\boldsymbol{Re}\, s > \sigma_0$
$u(t)e^{\sigma_0 t}cos\omega_0 t$ $\sigma_0, \omega_0 \in \mathbb{R}\ \sigma_0 > 0$	non esiste	$\frac{s-\sigma_0}{[(s-\sigma_0)^2+\omega_0^2]}$	$\boldsymbol{Re}\, s > \sigma_0$
$sin\omega_0 t$ $\omega_0 \in \mathbb{R}$	$\frac{\pi}{j}(\delta(\omega-\omega_0)-\delta(\omega+\omega_0))$	non esiste	
$cos\omega_0 t$ $\omega_0 \in \mathbb{R}$	$\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$	non esiste	
$\frac{sin\omega_0 t}{t}$ $\omega_0 \in \mathbb{R}$	$\pi(u(\omega+\omega_0)-u(\omega-\omega_0))$	non esiste	
$tu(t)$	$j\pi\delta'(\omega)-\frac{1}{\omega^2}$	$\frac{1}{s^2}$	$\boldsymbol{Re}\, s > 0$
$\frac{1}{2}t^2u(t)$	$\frac{j^2}{2}\pi\delta''(\omega)+\frac{1}{(j\omega)^3}$	$\frac{1}{s^3}$	$\boldsymbol{Re}\, s > 0$
$\frac{1}{(n-1)!}t^{n-1}u(t)$	$\frac{j^{n-1}}{(n-1)!}\pi\delta^{(n-1)}(\omega)+\frac{1}{(j\omega)^n}$	$\frac{1}{s^n}$	$\boldsymbol{Re}\, s > 0$
$u(t)e^{j\omega_0 t}\frac{t^{n-1}}{(n-1)!}$ $\omega_0 \in \mathbb{R}$	$\frac{j^{n-1}}{(n-1)!}\pi\delta^{(n-1)}(\omega-\omega_0)+\frac{1}{(j\omega-j\omega_0)^n}$	$\frac{1}{(s-j\omega_0)^n}$	$\boldsymbol{Re}\, s > 0$
1	$2\pi\delta(\omega)$	non esiste	
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	non esiste	
sgn$t$	$\frac{2}{j\omega}$	non esiste	
$\frac{1}{t}$	$-\pi j$ sgnω	non esiste	
$\lvert t\rvert$	$-\frac{2}{\omega^2}$	non esiste	
$\frac{1}{t^2}$	$-\pi\lvert\omega\rvert$	non esiste	
$e^{-\alpha\lvert t\rvert}$ $\alpha \in \mathbb{R}\ \alpha > 0$	$\frac{2\alpha}{\alpha^2+\omega^2}$	$\frac{2\alpha}{\alpha^2+s^2}$	$-\alpha < \boldsymbol{Re}\, s < \alpha$

Proprietà della trasformata Zeta

Proprietà	**Segnale**	**Zeta**
Linearità	$ax(n)+by(n)$	$aX(z)+bY(z)$
Traslazione di n_0	$x(n-n_0)$	$z^{-n_0}X(z)$
Moltiplicazione per a^n	$a^n x(n)$	$X\left(\frac{z}{a}\right)$
Moltiplicazione per n	$nx(n)$	$-z\,\frac{dX(z)}{dz}$
Convoluzione	$x(n)*y(n)$	$X(z)\cdot Y(z)$

Trasformata $\mathcal{Z}$ di alcune funzioni sugli interi

$\boldsymbol{x(n)}$	$\boldsymbol{\mathcal{Z}[x(n)]=X(z)}$	$\boldsymbol{dom_{\mathcal{Z}}X(z)}$
$\delta(n)$	1	$\forall z$
$u(n)$	$\frac{1}{1-\frac{1}{z}}$	$\|z\|>1$
$nu(n)$	$\frac{1/z}{\left(1-\frac{1}{z}\right)^2}$	$\|z\|>1$
$u(n+N)-u(n-N)$	$z^N\frac{1-z^{-2N}}{1-\frac{1}{z}}$	$\forall z$
$a^n u(n)$ $a\in\mathbb{C}$	$\frac{1}{1-\frac{a}{z}}$	$\|z\|>\|a\|$
$u(n)sin\omega n$	$\frac{\frac{1}{z}sin\omega}{\left(\frac{1}{z^2}\right)-\left(\frac{2}{z}\right)cos\omega+1}$	$\|z\|>1$
$u(n)cos\omega n$	$\frac{1-\frac{1}{z}cos\omega}{\left(\frac{1}{z^2}\right)-\left(\frac{2}{z}\right)cos\omega+1}$	$\|z\|>1$
$u(n)e^{-\sigma n}sin\omega n$	$\frac{\frac{1}{z}e^{-\sigma}sin\omega}{\left(\frac{1}{z^2}\right)e^{-2\sigma}-\left(\frac{2}{z}\right)e^{-\sigma}cos\omega+1}$	$\|z\|>e^{-\sigma}$
$u(t)e^{-\sigma n}cos\omega n$	$\frac{1-\frac{1}{z}e^{-\sigma}cos\omega}{\left(\frac{1}{z^2}\right)e^{-2\sigma}-\left(\frac{2}{z}\right)e^{-\sigma}cos\omega+1}$	$\|z\|>e^{-\sigma}$

Scomposizione in fratti semplici

$F(s) = \frac{P_n(s)}{Q_n(s)}$

- ***Poli semplici:***
  $F(s) = Q_{n-m}(s) + \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} + \cdots + \frac{A_m}{s-s_m}$
  dove $A_i = Res_{F(s)}(s_i)$, con $i = 1,2,\dots,m$.
  Quindi $F(s) = Q_{n-m}(s) + \frac{Res_{F(s)}(s_1)}{s-s_1} + \frac{Res_{F(s)}(s_2)}{s-s_2} + \cdots + \frac{Res_{F(s)}(s_m)}{s-s_m}$.
- ***Poli multipli:***
  $F(s) = Q_{n-m}(s) + \frac{A_m}{(s-s_0)^m} + \frac{A_{m-1}}{(s-s_0)^{m-1}} + \cdots + \frac{A_2}{(s-s_0)^2} + \frac{A_1}{(s-s_0)^1}$
  dove $A_i = \frac{1}{(i-1)!}\frac{d^{(i-1)}}{ds^{i-1}}(s-s_i)^{m-i}\frac{P_n(s)}{Q_m(s)}$.
- ***Poli complessi coniugati:***
  - ***Poli semplici complessi coniugati:***
    Data una coppia di poli semplici complessi coniugati
    $s_0 = \sigma_0 + j\omega_0 \qquad \overline{s_0} = \sigma_0 - j\omega_0$
    anche i residui della funzione in corrispondenza dei suddetti poli sono complessi coniugati.
    Preso
    $Res_{F(s)}(s_0) = \alpha + j\beta$
    residuo di $F(s)$ in s_0, la scomposizione in fratti semplici sarà del tipo:
    $F(s) = Q_{n-m}(s) + 2\alpha\frac{s-\sigma_0}{(s-\sigma_0)^2+\omega_0^2} - 2\beta\frac{\omega_0}{(s-\sigma_0)^2+\omega_0^2}$.
    Chiaramente, risulta necessario il calcolo di un unico residuo indifferentemente in s_0 o in $\overline{s_0}$.
    $\alpha = \Re[Res_{F(s)}(s_0)] \qquad \beta = \Im[Res_{F(s)}(s_0)]$
    $\alpha = \Re[Res_{F(s)}(\overline{s_0})] \qquad \beta = \Im[Res_{F(s)}(\overline{s_0})]$.
  - ***Poli multipli complessi coniugati:***
    Data una coppia di poli semplici complessi coniugati
    $s_0 = \sigma_0 + j\omega_0 \qquad \overline{s_0} = \sigma_0 - j\omega_0$
    presi del secondo ordine, si valutano i seguenti residui:
    $\alpha + j\beta = Res_{F(s)}(s_0)$
    $\alpha + j\beta = Res_{(s-s_0)F(s)}(s_0)$.
    La scomposizione in fratti semplici avrà la forma:
    $F(s) = Q_{n-m}(s) + 2\alpha\frac{s-\sigma_0}{(s-\sigma_0)^2+\omega_0^2} - 2\beta\frac{\omega_0}{(s-\sigma_0)^2+\omega_0^2} - \frac{d}{ds}\left[2\alpha\frac{s-\sigma_0}{(s-\sigma_0)^2+\omega_0^2} - 2\beta\frac{\omega_0}{(s-\sigma_0)^2+\omega_0^2}\right]$
    dove vale, come visto in precedenza
    $\alpha = \Re[Res_{F(s)}(s_0)] \qquad \beta = \Im[Res_{F(s)}(s_0)]$
    $\alpha = \Re[Res_{(s-s_0)F(s)}(s_0)] \qquad \beta = \Im[Res_{(s-s_0)F(s)}(s_0)]$.

Calcolo integrali con la teoria dei residui

Un integrale espresso nella forma $\int_{-\infty}^{+\infty}$ è detto *integrale di Cauchy*.

$$\int_{-\infty}^{+\infty}\frac{P(t)}{Q(d)}e^{j\alpha t}\,dt = \begin{cases} 2j\pi\left[\sum_{k=1}^{n}Res_{f(t)}(s_k) + \frac{1}{2}\sum_{i=1}^{m}Res_{f(t)}(\overline{s_i})\right] & \text{se } \alpha > 0 \\ -2j\pi\left[\sum_{k=1}^{n}Res_{f(t)}(s_k) + \frac{1}{2}\sum_{i=1}^{m}Res_{f(t)}(\overline{s_i})\right] & \text{se } \alpha < 0 \end{cases}$$